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Analysis of mathematical connection in abstract algebra

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Abstract. This research aims to improve students' understanding of the Concept Group in instructions of Abstract Algebra through mathematical connection analysis. This research was conducted at the University of Papua for 32 student's mathematics student who took the course of the algebraic structure in the academic year 2017/2018. Instruction was carried out using cooperative model, through the application of group investigation. Students were grouping based on the prerequisite lesson grades, especially introduction to mathematics. The results show that the connection between concepts provides better results on improving students' concept of understanding, compared to conventional teaching learning, where lecturers provide a formal definition, provide further examples of theorem and proof of theorem.

1. Introduction

Mathematics is a unique subject and is a fundamental part of the school curriculum [1]. This is an instrument for the development of all other sciences. Consciously or unconsciously, we use mathematics in every aspect of life [2]. However, the majority of students around the world do not like mathematics. In addition, learning mathematics in schools is different from at university. Learning mathematics at university is more difficult because concepts are studied abstractly [3] The subject which are considered as difficult subject by students because of their abstractness are lesson in the abstract algebra group such as algebraic structure.

On the other hand, the algebraic structure is a distinguished lecture at the university, especially for mathematics study programs and mathematics education study programs. In the study program of mathematics education at the University of Papua, the algebraic structure is a subject that must be taken by all students [3, 4]. However, some previous studies have reported that students' understanding of abstract algebra is less satisfying [5, 6]. The principal thing that makes students had difficulties in studying the course is its characteristics.

The characteristics of the algebraic structure have a strict and coherent axiomatic deductive structure, loaded with abstract concepts in both definitions and theorems [7] An algebraic structure is understood to be an arbitrary set, with one or more operations defined on it [8]. So, the algebraic structure is any set with one or more operations defined in the set. Furthermore, in an algebraic structure, elements of a set connected by an operation must fulfil certain axioms [9]. These axioms define concepts such as Group, Vector Space, Ring, Field, Lattice and Module in the subject of algebraic structures.

In this study will highlight the simple algebraic structure of the Group. A Group is defined as nothing that can be used for an action that allows it to be used, as an identity, has an identity element and has an inverse for each set element. The theory group is a formal definition, theorem, and proof



[10]. The purpose of group theory learning is that students are able to solve problems using abstract definitions, skilled at manipulating symbolic and proving group theorems.

The author's experience during teaching algebraic structures, specifically the concept of the Group, is very difficult to get the precise teaching models and methods that can improve student learning outcomes. Group concept learning generally uses conventional learning. Lecturers provide a formal definition, provide further examples of theorem and proof of theorem. The learning step like that has been widely used in teaching algebraic structures, especially the concept of the Group. Students' understanding is limited to what the lecturers receive in class. They understand the definition of the group and its theory just to memorize. Even with examples and evidence, everything is memorized. Students have difficulty giving examples and not examples of group definitions.

The difficulty of students in understanding the abstract concept in the theory group is because students are not able to make connections between ideas and concepts of group theory with their prior knowledge [11]. Students are less able to connect old knowledge with their new knowledge [7]. This connection will occur if students are able to extract themselves a new concept based on what they knew before [3]. By doing the connection, the mathematics concepts which have been learned can contribute as strong foundation in comprehending a new concept [12].

The connection process will occur maximally when students work in groups. One method that involves students learning in groups is the cooperative learning model. Cooperative learning model consists of various types, one of which is type investigation group.

2. Methods

This research was conducted at the study program of mathematics education faculty of teacher training and education at University of Papua. The subject of this research is 32 of the mathematics education students. They are the student who took the lecture of the algebraic structure in the academic year 2017/2018.

The research was carried out using descriptive methods. The descriptive method is a research method that has the purpose of gathering information about the present existing condition [13]. The current situation used is the learning outcome, especially about the student conceptual understanding of group theory. Measurement of students' understanding of concepts using test instruments. Tests are used in the form of essays, so that the answers presented by students are more objective, according to what they know.

Instruction was carried out using cooperative model, through the application of group investigation. The learning model is a cooperative learning model that requires students to be active and participate in the learning process by digging / searching for information / material to be studied independently with available materials [14].

The implementation stage of the learning model begins by guiding students to form a group and identify topics. Then students plan the tasks to be studied and carry out investigations. After the results of the investigation have been obtained, each group prepares a final report to later be presented and evaluated together [15].

3. Result and discussion

The process of mathematical connections in learning group theory starts from concrete ideas that are used as motivation to define an abstract concept. The main focus of group theory at the beginning of learning is how students understand the ideas and concepts of abstract group definitions. Furthermore, students are expected to understand the axioms in the Group structure that form the basis for other structures in abstract algebra classes.

Algebraic learning activities begin with the motivational question "what do you know about the GROUP?" Individual spontaneous responses to this question get two interesting answers: First answer: a group is a group of objects.

Second answer: a group is grouping things with clear goals and conditions.

These two answers are enough to build a mathematical connection to abstract group definitions. The researcher then continues learning by dividing students into four groups. Each group was given investigative material concerning 2 (two) abstract objects that were very well understood by students, namely:

1. Set of integers, $Z = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$, with addition dan multiplication operation.
2. Set of Matrices 2×2 , $M = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid a, b, c, d \in R \right\}$, with matrix addition and multiplication operations.

Based on these two abstract objects, researchers investigated the properties related to addition and multiplication operations. These characteristics are closeness, associative, identity and inverse. The results of the investigation of 4 groups provide conclusions:

1. For sets of integers $Z = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$, the sum operation satisfied the properties: closed, associative, has zero identity elements and each Z member has inverse. For multiplication operations, Z is closed, associative, has an identity element 1 but has no inverse. The inverse existence of Z multiplication is obtained from the results of the investigation of Group K1

K1
ambil $3, x \in Z$
 $3 + 0 \in Z$, ada $-3 \in Z$
identitas $-3 + 3 = 0$
Untuk perkalian
 $3 \in Z$
 $3 \times \dots = 1$ identitas
 $3 \times \frac{1}{3} = 1$ identitas
 $\frac{1}{3} \notin Z$ gagal.

Figure 1. The investigation of group K1

2. Likewise, matrices $M = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid a, b, c, d \in R \right\}$, on the matrix sum operation, the matrix M is closed, associative, has an identity element, $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. The M matrix will have an inverse if $\det(M) \neq 0$. This conclusion is based on the results of the K3 group investigation.

K3 $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid a, b, c, d \in R$
Invers matriks $= \frac{1}{\det M} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} = \frac{1}{ad-bc} \neq 0$
Invers M ada kalau $ad-bc \neq 0$

Figure 2. The investigation of group K3

Students are then guided to understand the set of Z integers for the sum operation to form a system. The same is true for the M Matrix with Real elements. In this matrix the addition operation will form a system if the determinant value is not equal to zero. In addition, there are other sets that form a particular system for certain operations. Mathematical connections occur when students are able to abstract the Z set system to any non-empty set G and the sum operation of a binary operation $*$. In this process a new concept is formed which is known as the Group concept.

Group theory learning through mathematical connection analysis by linking ideas between concepts can increase the number of students who understand the concept of the group better than students' understanding if not using the connection analysis. There were 65% of students who were able to construct examples from the Group and 55% of students were able to identify any association with an operation as a group or not a group.

4. Conclusion

Based on the research can be concluded that using the connection between concepts provides better results on improving students' concept of understanding, compared to student understanding on conventional teaching-learning. On conventional teaching of abstract algebra, lecturers provide a formal definition, provide further examples of theorem and proof of the theorem.

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