DATA ARTIKEL ILMIAH

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Nama Jurnal	:	Open journal of Advanced Engineering Techniques
Volume/Nomor	:	Volume 2, No 3
Edisi	:	April 2014
Alamat web	:	https://www.researchgate.net/publication/271030322_An_Implemented_Software_for_Transient_Stability_Analysis_of_SMIB_Based_on_Runge-Kutta_Method
Kategori	:	Jurnal Internasional berbahasa PBB
SJR	:	-

<u>Kindi Publication</u>

Open Journals of Technical Sciences

Advisory/Reviewers Board

Open Journal of Advanced Engineering Techniques OJAET
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www.kindipublication.com
Vol.2, No.3, 1-6
April, 2014
ISSN: 2336-0062

An Implemented Software for Transient Stability Analysis of SMIB Based on Runge-Kutta Method

Adelhard Beni Rehiara and Sabar Setiawidayat

Abstract**—**Single machine infinity bus (SMIB) will be a simple way to examine a complex electrical power system. In investigating the transient stability of a SMIB system, equal area criterion (EAC) method can be used to get critical clearing angle δ_{cr} and critical clearing time t_{cr}. In each case of a SMIB, critical clearing time t_{cr} cannot directly be determined using equal area criterion method. This paper will introduce Runge-Kutta method utilized to modify the critical clearing time t_{cr} found with EAC method and to know the best time to clear a fault. In this case, the critical clearing time t_{cr} of EAC method is almost same for every fault and it is faster than the critical clearing time t_{cr} of Runge-Kutta method. A software package was designed to reduce and fasten the routine of calculation. Overall simulation shows that the designed software has worked properly.

Keywords**—** Transient stability, single machine infinity bus, equal area criterion, Runge-Kutta, step by step.

I. INTRODUCTION

ELECTRICAL power systems consist of generation, transmission and distribution system and/or also load as the user of the electrical power. The other components that can probably be connected to the systems are transformer, circuit breaker, relay protection, prime mover, etc. All of the power system components are used to maintain the quality, continuity, stability and reliability of the systems [12].

SMIB (single machine infinite bus) is a simple electrical power system that has a generator connected to infinite bus as load [11]. To make it simple to be analyzed, an interconnection electrical power system can be separated into some SMIBs [12].

Stability is an important constraint in power system operation [2]. The major problem in every electrical power system is how stability of the system when a fault happens for example short circuit, broken line, disconnected load, etc. Equal area criterion (EAC) is a classic method in transient stability that is applicable to all two machines systems (SMIB). The method provides an easier way to determine the critical clearing angle δ_{cr} but the method does not able to find directly the critical clearing time t_{cr} [4],[13]. Another mathematical calculation, called Runge-Kutta methods, will be used to solve the limitation of the EAC methods [12].

Khan (1998) had designed software for transient stability analysis based on EAC. This software is focused on EAC method and also provided a graphical utility for simulating power angle swing. The software gets clearing time t_c based on an input of clearing angle δ_c.

This paper will use both EAC and Runge-Kutta methods to get the best critical clearing time t_{cr}. Then both methods will be implemented in a software package.

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II. TRANSIENT STABILITY

A. Equal Area Criterion

The Equal area criterion method is based on an assumption that acceleration area A_1 should be same with deceleration area A_2 (figure 1). The areas are separated between normal power angle δ and maximum power angle δ_{max} equally by critical clearing angle δ_{cr}. A power system will be stable if the acceleration area A_1 is smaller or equal to the deceleration area A_2. According to the assumption, critical clearing angle δ_{cr} and maximum power angle δ_{max} can be found as follow [10-13].

$$\delta_{cr} = \cos^{-1}\left[\frac{Pm(\delta_{\max} - \delta) - P\max_{II}\cos\delta + P\max_{III}\cos\delta_{\max}}{P\max_{III} - P\max_{II}}\right] \quad (1)$$

$$\delta_{\max} = \pi - \sin^{-1}\left(Pm / P\max_{III} \right) \quad (2)$$

The critical clearing time also can be established in EAC method as shown in equation 3.

$$t_{cr} = \sqrt{\frac{2H(\delta_{cr} - \delta)}{\pi f\, P_m}} \quad (3)$$

Where H is the stored kinetic energy at synchronous speed per MVA base, f is the frequency in hertz, P_m is the shaft

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April, 2014
ISSN: 2336-0062

power input less rotational losses in pu, t is the time in second and P_{max} is the maximum electrical power in pu while the subscript I, II and III indicate the condition of steady state (pre), during and post fault.

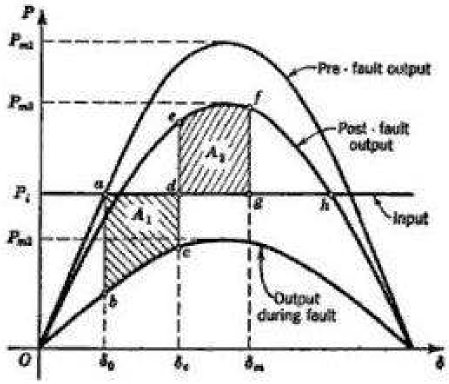

Fig. 1 Power curve

B. Runge-Kutta Method

The Runge-Kutta method is a mathematical method to solve the differential equation in every step of time (step by step). The step by step calculations will be used to find critical clearing time t_{cr} according to critical clearing angle defined with the EAC method. The power angle δ is calculated for several times to ensure that the angle δ will be oscillated indicating that the system is stable.

Curve δ versus t of a machine should be made to investigate the swing curve of a machine. By determining the swing curve in various clearing time, the maximum time permitted to clear the fault can be determined.

Some numeric methods, which are often used to solve the differential equation of step by step calculation, are the methods of Euler, Heun, Runge-Kutta, etc. In this case, it will be focused on the method of Runge-Kutta.

Fourth order of Runge-Kutta method can be utilized to analyze the swing equation. The equation can be rewritten as [3],[6],[8],[12]:

$$\frac{d\delta}{dt} = \omega - \omega_s$$

$$\frac{d\omega}{dt} = \frac{\omega}{2H}\left(P_m - P_e\right) = \frac{\omega}{2H}P_a \quad (4)$$

Where Pe is the electrical power in pu, Pa is the accelerating power in pu and ω is the angular displacement of the rotor in rad. By substituting the swing equation to the method of Runge-Kutta, four estimations can be obtained [6].

First estimation:

$$k_{1i} = \left.\frac{d\delta}{dt}\right|_{(t)} \Delta t = \left(\omega_{(t)} - \omega_s\right)\Delta t \quad (5)$$

$$l_{1i} = \left.\frac{d\omega}{dt}\right|_{(t)} \Delta t = \frac{\pi f}{H}\left(Pm - Pe_{(t)}^{(1)}\right)\Delta t \quad (6)$$

Second estimation:

$$k_{2i} = \left\{\left(\omega_{(t)} + \frac{l_{1i}}{2}\right) - \omega_s\right\}\Delta t \quad (7)$$

$$l_{2i} = \frac{\pi f}{H}\left(Pm - Pe_{(t)}^{(2)}\right)\Delta t \quad (8)$$

Third estimation:

$$k_{3i} = \left\{\left(\omega_{(t)} + \frac{l_{2i}}{2}\right) - \omega_s\right\}\Delta t \quad (9)$$

$$k_{3i} = \left\{\left(\omega_{(t)} + \frac{l_{2i}}{2}\right) - \omega_s\right\}\Delta t \quad (10)$$

Fourth estimation:

$$k_{4i} = \left\{\left(\omega_{(t)} + l_{3i}\right) - \omega_s\right\}\Delta t \quad (11)$$

$$l_{4i} = \frac{\pi f}{H}\left(Pm - Pe_{(t)}^{(4)}\right)\Delta t \quad (12)$$

Where i = 1, 2, 3, ... , n and in the end of the period, the power angle δ and the synchronous speed ω will be changed using both equations below.

$$\delta i_{(t+\Delta t)} = \delta_{(t)} + \tfrac{1}{6}\left(k_{li} + 2k_{2i} + 2k_{3i} + k_{4i}\right) \quad (13)$$

$$\omega_{(t+\Delta t)} = \omega_{(t)} + \tfrac{1}{6}\left(l_{li} + 2l_{2i} + 2l_{3i} + l_{4i}\right) \quad (14)$$

The calculation will be continued for $t=t+\Delta t$ until the required duration of time. Critical clearing time t_{cr} can be estimated using a linear interpolation method, which is formulated as [1][12]:

$$t_{cr} = t_c + \frac{\tfrac{1}{2}\left(\delta_{c+1} + \delta_c\right) - \delta_c}{\delta_{c+1} - \delta_c}\Delta t \quad (15)$$

Where t_c and t_{c+1} are the time for clearing and after clearing fault while δ_c and δ_{c+1} are the angle at clearing and after clearing fault.

III. CASE STUDY

The single line diagram of an investigated power system is shown in the figure 2. The system consists of a generator attached to two of step up transformers and step down transformers connected to infinite bus, two transmission lines and a load. Rehiara (2009) had worked for this case study as follows.

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ISSN: 2336-0062

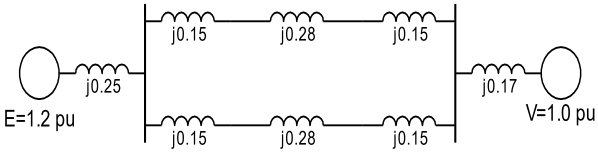

Fig. 2 Single line diagram

A. EAC Calculations

The EAC calculations will be separated into pre, during and post fault condition.

Pre fault

To get the electrical power *Pe*, a transfer reactance *Xt*, which is the reactance from the generator to the load, should be known. From the figure 2, the transfer reactance *Xt* can be calculated as follows:

$$Xt = j0.25 + \frac{j(0.15+0.28+0.15)\,j(0.15+0.28+0.15)}{j(0.15+0.28+0.15)+j(0.15+0.28+0.15)} + j0.17$$
$$= j0.71$$
$$Pe = \frac{|V||E|}{Xt}\sin\delta = \frac{1.2}{0.71}\sin\delta$$
$$= 1.69014\sin\delta$$

With shaft power input *Pm* is 1.0 pu, the steady state power angle δ is:

$$\delta = \sin^{-1}(1/1.69014)$$
$$= 36.2754° = 0.6331\,\text{rad}$$

During fault

Because the reactance of step up and step down transformers and also transmission lines are same, the calculation of during fault can only be done for line 1 or 2.

The reactance system after Δ-Y conversion is shown in figure 3. The transfer reactance of each fault is found by converting it back with Y-Δ conversion. The transfer reactance and maximum power of each fault are shown in table I.

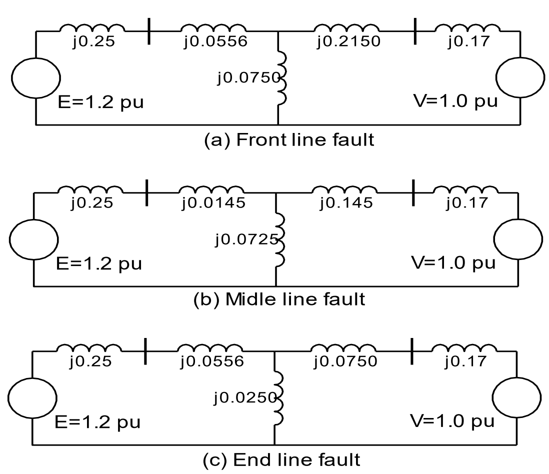

Fig. 3 Δ-Y conversion

Post fault

When the fault is cleared, the system is operated by using a line. So the transfer reactance *Xt* and electric power *Pe* will be changed and those can be calculated as:

$$Xt = j0.25 + j0.15 + j0.28 + j0.15 + j0.17$$
$$= j1.0$$
$$Pe = 1.2\sin\delta$$

The maximum power angle $\delta_{\max}$ and the critical clearing angle are:

$$\delta_{\max} = \pi - \sin^{-1}\left(1/1.2\right)$$
$$= 2.1565\,\text{rad}$$

$$\delta_{cr} = \cos^{-1}\left[\frac{1(2.1565-0.6331)-0.4054\cos 0.6331+1.2\cos 2.1565}{1.2-0.4054}\right]$$
$$= 0.835191\ \text{rad} = 47.8529°$$

In EAC method, the critical clearing time t_{cr} is offered by equation 3 and the result for fault in front line is:

TABLE I
VARIABLES DEFINED WITH EAC METHOD

Variables	Fault Location		
	Front Line	Middle Line	End Line
Xt (pu)	j2.960	j2.4262	j2.759
Pmax (pu)	0.4054	0.4946	0.4349
δcr (deg)	47.8529	49.1602	48.2543
tcr (s)	0.101	0.107	0.103

$$t_{cr} = \sqrt{\frac{8(47.8529-36.2764)}{50\pi}}$$
$$= 0.101s$$

Power angle δ of normal operation and maximum power Pmax of pre and post fault are same wherever the fault happens. Result of calculations for the EAC method is given in table I.

TABLE II
RESULT OF CLEARING TIME

Clearing Time (s)	Fault Location		
	Front Line	Middle Line	End Line
0.05	Stable	Stable	Stable
0.10	Stable	Stable	Stable
0.15	Not stable	Stable	Not stable
0.20	Not stable	Not stable	Not stable
0.30	Not stable	Not stable	Not stable

B. Step by Step Calculations

The system is assumed to work with frequency 50 Hz and

H=4MJ/MVA and the iteration interval is 0.05s. For the first time the fault happens, the acceleration power Pa is not in synchronous. So the acceleration power Pa is the average of Pa pre and Pa during fault. The step by step calculation was done in Matlab with several clearing times t_c and it was presented in following table.

Base on table II, the best clearing time t_c for fault in front and end lines are in between 0.10 and 0.15s. For fault in the middle line, the best clearing time will be in between 0.15 and 0.20s.

TABLE III
RESULTS OF THE STEP BY STEP CALCULATIONS

t (s)	Front line fault		Middle line fault		End line fault	
	Δδ (deg)	δ (deg)	Δδ (deg)	δ (deg)	Δδ (deg)	δ (deg)
0-	-	36.2754	-	36.2754	-	36.2754
0+	-	36.2754	-	36.2754	-	36.2754
0avg	0.9931	36.2754	0.9931	36.2754	0.9931	36.2754
0.05	4.2376	37.2685	3.9342	37.2685	4.1372	37.2685
0.10	8.3428	41.5061	7.7167	41.2027	8.1351	41.4057
0.15	12.2213	49.8488	11.2400	48.9193	11.8950	49.5408
0.20	15.8361	62.0701	14.4556	60.1594	15.3758	61.4358
0.25	19.2465	77.9062	17.4124	74.6150	18.6340	76.8116
0.30	22.6354	97.1527	20.2825	92.0274	21.8501	95.4456
0.35	26.3170	119.7880	23.3691	112.3098	25.3371	117.2956
0.40	30.7078	146.1051	27.0917	135.6790	29.5167	142.6328
0.45	36.2269	176.8129	31.9284	162.7707	34.8341	172.1495
0.50	43.0643	213.0398	38.2611	194.6991	41.5487	206.9836

To find the critical clearing time t_{cr}, every fault will be catch for a few milliseconds and find the critical clearing time t_{cr} by checking the critical clearing angle δ_{cr} in each step of time duration.

The results of the step by step calculations are shown in table III. The results are only provided the data for a half of first second calculations. The shaded areas inside the table indicate that the critical clearing time t_{cr} is probably in between the step.

For fault in front line, the critical clearing angle t_{cr} is about 47.8529 degree and it is in between step 0.1s and 0.15s (see table III). The critical clearing angle t_{cr} can be estimated by using the equation 15 with data provided in table III and it can be calculated as follows.

$$t_{cr} = 0.10 + \frac{\frac{1}{2}(48.0217 + 41.3577) - 41.3577}{48.0217 - 41.3577} x0.05$$
$$= 0.125s$$

With the same method, the critical clearing time t_{cr} for fault in the middle and the end line are found about 0.175s and 0.125s.

IV. RESULT

A. Flowchart

This software design is focused on time domain system which will find critical clearing time t_{cr} based on critical clearing angle δ_{cr} found with EAC method. Therefore the flow chart shows the step by step calculation using Runge Kutta method. The flow chart using for programming the software especially for data processing is shown in figure 4.

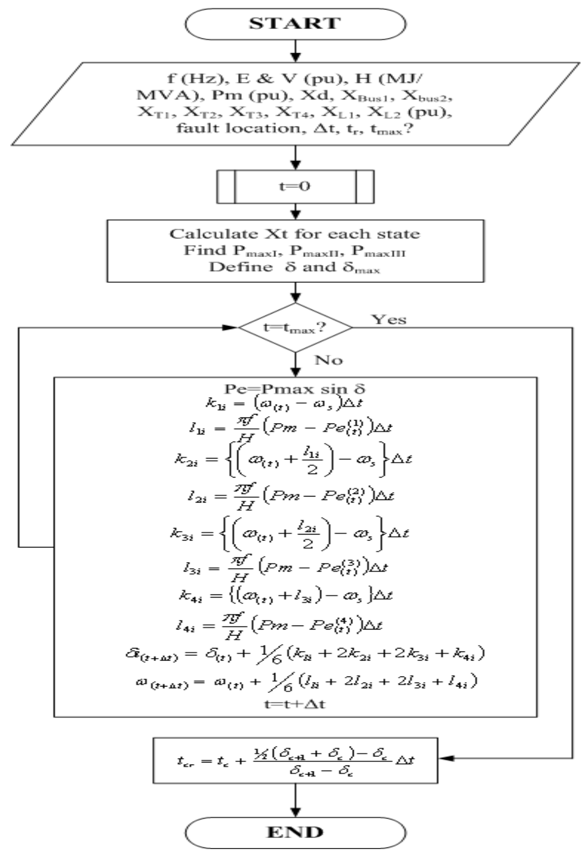

Fig. 4 Flowchart

B. Software Design and Simulation

The software was implemented using Borland Delphi. Some features of the software are for data entry and edit data, for graphs of rotor angle, speed, acceleration, single line and EAC graphs, and for tables of step by step and power. Output data can be shown both graphics and tables also the output data can be printed out. The result of software simulations using the data from case study were shown in fig. 5, 6 and 7.

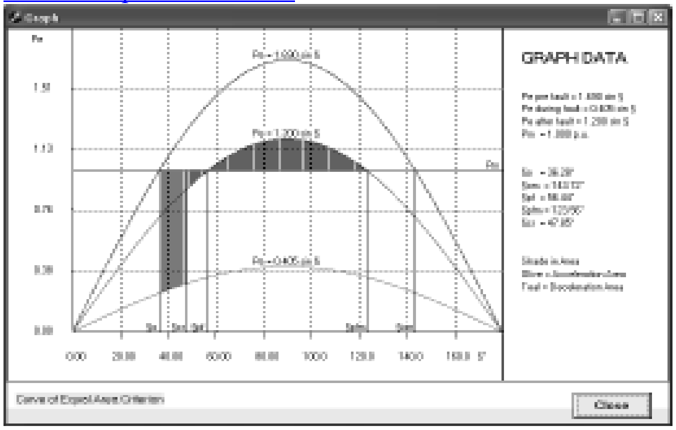

Fig. 5 Curve of EAC

Power curves of the system are shown in figure 5 and the power curve of pre, during and post faults are 1.69 sin δ, 0.405 sin δ, and 1.2 sin δ. Acceleration area in left shaded is smaller than deceleration area in right shaded means that system is stable.

Curve δ-t with clearing time tc about 0.1s is shown in figure 6 by giving same data input as in manual calculation. Critical clearing time t_{cr} which is defined using eq.15 is found in between 0.1s and 0.15 about 0.125s. It is mean that the system will become unstable if the clearing time t_{cr} is bigger than 0.1s. The simulation demonstrates big swing of the rotor angle δ because the clearing time t_c is close to the critical clearing time t_{cr}.

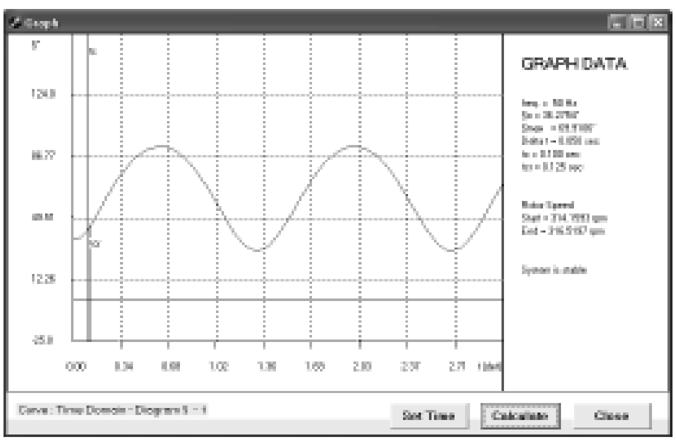

Fig. 6 Curve of δ

With the same clearing time t_c, the curve of ω-t (fig.7) also demonstrate the relation with the rotor angle δ. After the disturbance, the rotor speed ω is increase proportionally with the rotor angle δ. The different of the both rotor property is that decreasing of rotor speed is growth faster and after that it is slowly down; on the other hand the rotor angle δ tends to be oscillating in stable way.

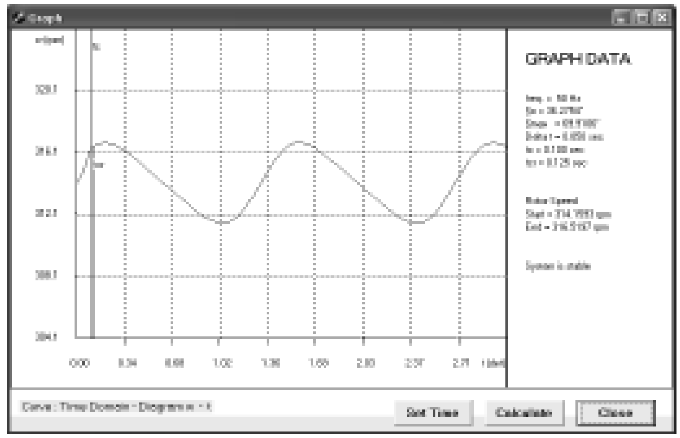

Fig. 7 Curve of ω

The critical clearing angle δ_{cr} about 47.8529 is placed in between 0.1s and 0.15s (table III). Therefore by using the linear interpolation method, the critical clearing time t_{cr} is found about 0.125s. To get an accurate critical clearing time t_{cr}, the step of time can be set lower than 0.05s. An investigation with step time about 0.0001s demonstrates that system is still stable in clearing time 0.1172s with rotor angle at this moment about 47.8532°. The system will be out of stable if the fault is cleared at 0.1173s with clearing angle about 47.9727°. On the other hand for fault in the middle and the end lines, the best clearing times are 0.1286s and 0.1208s for clearing angle about 49.1584° and 48.2617°.

V. Conclusion

Critical clearing time t_{cr} found with EAC method is almost same for the investigated system wherever the fault happens. For each fault in the system, the critical clearing angle δ_{cr} is different because the electrical power *Pe* for every fault is different. The difference also makes an influence for the critical clearing time t_{cr}.

Using Runge-Kutta method, critical clearing angle δ_{cr} can not be used to define the critical clearing time t_{cr} because the calculation using step by step is never exactly match with the critical clearing angle δ_{cr}. In this case, the critical clearing time t_{cr} can be found with a linear interpolation method.

The simulation result shows that the critical clearing time t_{cr} defined with EAC method is faster than with Runge-Kutta method. Some differences appear in the result but both EAC and Runge-Kutta methods prove that the critical clearing time t_{cr} is longer for every fault in the middle line.

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